41 - Fluency in single-digit addition and related subtraction facts among Indian students

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Fact fluency and automaticity: Addition and subtraction facts fluency is the ability to compute a single-digit addition fact or the related subtraction fact in 3 seconds or less

Objective of the study and research questions

a) What are the measures to measure the levels of fluency in single-digit addition facts and related subtraction facts?

b) What are the levels of fluency in single-digit addition facts and related subtraction facts? By what grades students develop automaticity?

c) Are there significant differences in the levels of fluency among government schools and high-fee English-medium private schools?

Motivation for the study

- Do students especially in govt. schools struggle in learning standard algorithms to add, subtract, multiply and divide whole numbers due to lack of fact fluency?
- As per the existing research literature, the retrieval deficit of arithmetical basic facts can be said to be a useful indicator in the diagnosis of learning difficulties in mathematics
- Do students demonstrate student-invented strategies described in existing research literature in studies outside India? (Traditional classroom practices don’t focus on promoting this.)
- How do fact fluency develop? What are the different stages a learner maybe in? What are the indicators of fact fluency?

METHOD AND MEASURES

Participants: 560 students in each of grades 3, 4 and 5 in govt. schools and 60 students each in these grades from high-fee English-medium private schools in the primary study

Number of schools covered: 48 **Grades/Classes**: 3, 4, 5

Places: Solan district, Himachal Pradesh; Udaipur district, Rajasthan; Ahmedabad, Gujarat

Type of schools: Govt. schools and high-fee English medium private schools

Mode of selection: randomly picked from a class room (e.g. odd numbered serial numbers in attendance register;
Time: Beginning of the academic year (around 0 – 3 months)

Measures:

7 + 1 = __	4 + 2 = __	2 + 3 = __	2 + 5 = __
5 + 4 = __	3 + 4 = __	1 + 7 = __	3 + 7 = __

5 – 1 = __	8 – 1 = __	5 – 2 = __	9 – 3 = __
6 – 2 = __	10 – 3 = __	7 – 2 = __	7 – 4 = __

- 20 addition facts with 10 each on a paper presented to a test taker
- Sums up to 5, 10 and 18 in increasing order of difficulty by large; also involves facts like 6 + 6 (doubling)
- subtraction facts related to the addition facts in the measures
- **Timed test**: 2 minutes given to verbally answer as many facts as a test taker can one-by-one in sequence
- An evaluator records answers and time remaining if any as test is conducted in one-to-one interactions
- Adapted from early grade mathematics assessment (EGMA) toolkit by the Research Triangle International

RESULTS

	Grade 3		Grade 4		Grade 5	
Percentile	**Govt.**	**Pvt.**	**Govt.**	**Pvt.**	**Govt.**	**Pvt.**
10th	0	9.2	3.5	13.4	5.5	13.6
25th	2.5	11.5	6.5	17.5	8.0	21.1
50th	6.5	15.0	9.5	22.6	10.9	28.6
75th	9.7	21.9	13.1	30.2	14.8	34.7
90th	13.4	29.0	17.1	37.5	19.4	41.5

Table 1: Learning levels on Addition Fact Fluency: Number of facts answered correctly in a minute (ncpm)

Fluency with addition facts (fluent if ncpm ≥ 20):

- **Private schools:** ≥ 75 percentile students in grade 3, above 50th percentile in grade 4 and above 25th percentile in grade 5
- **Govt schools**: even the students above 90th percentile were not fluent in both addition and subtraction facts in grades 3, 4 and 5 (except a few in grade 5 in addition facts).

	Grade 3		Grade 4		Grade 5	
Percentile	**Govt.**	**Pvt.**	**Govt.**	**Pvt.**	**Govt.**	**Pvt.**
10th	0.0	7.5	0	11.0	1.5	11.7
25th	0.0	8.5	4	13.3	5.5	16.0
50th	4.0	11.9	6.5	19.3	8.0	21.4
75th	6.5	17.6	9	23.8	10.5	33.1
90th	8.0	20.4	11.6	32.8	12.7	37.5

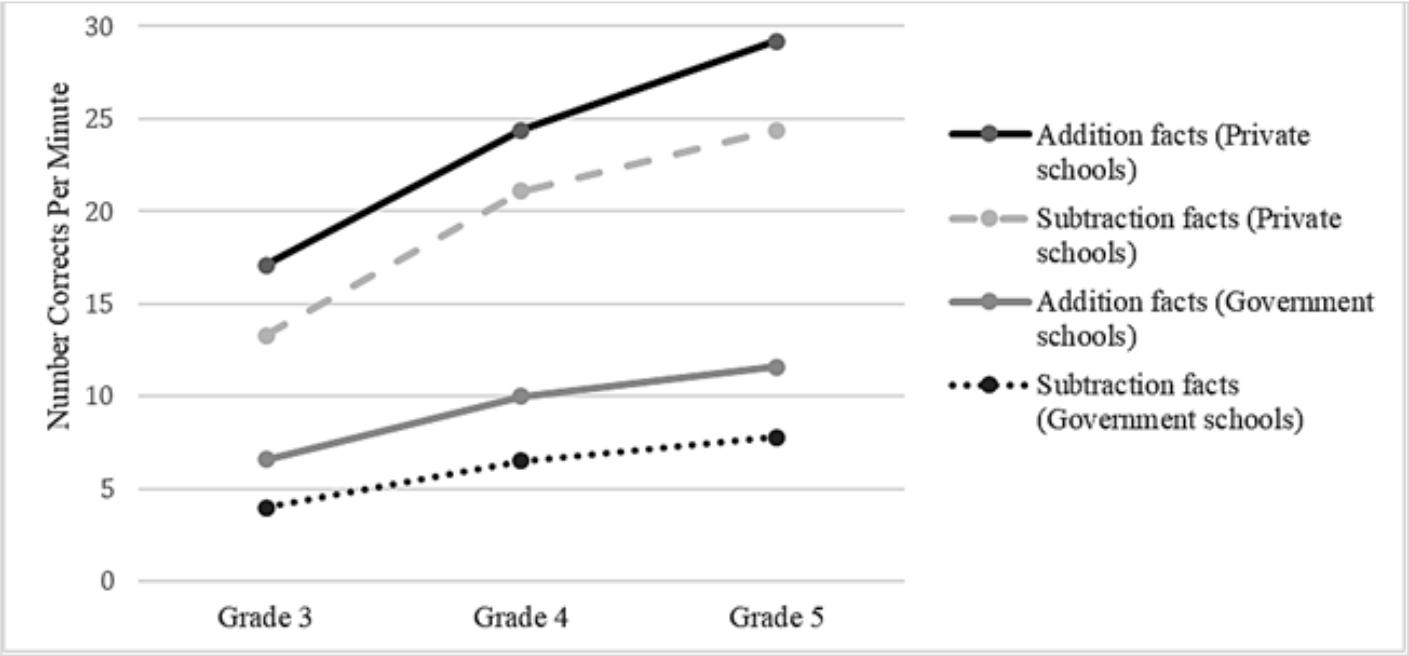
Table 2: Learning levels on Subtraction Fact Fluency: Number of facts answered correctly in a minute (ncpm)

Fluency with subtraction facts (fluent if ncpm ≥ 20):

- **Private schools:** Above 75 percentile students in grade 4, above 50th percentile in grade 5
- **Govt schools**: even the students above 90th percentile were not fluent in grades 3, 4 and 5

RESULTS AND DISCUSSION

- Considerable difference in fact fluency between students in English-medium private schools and govt. schools in both addition and subtraction facts in each of the grades 3, 4 and 5
- Fact fluency do increase going higher up the grades; the improvements are more in private schools going up the grades and hence the gap between the levels of fluency in private schools and government schools widened moving up the grades from 3 to 5. The gap does not close by grade 5.
- Fact fluency in subtraction gets developed later than addition



Zero scores:

- **Govt. schools**: 16.3% and 32.1% of grade 3 students did not answer any of the addition and subtraction facts correctly
- 11.81% of grade 4 and 7.82% of grade 5 government school students did not answer any of the subtraction facts correctly.
- **Private school**: There was no student in a private school, who was not able to answer even a single item correctly.

Learning levels: Figure: Average number of addition and subtraction facts answered correctly in a minute across grades 3, 4 and 5

Results – Strategies to answer an addition or subtraction fact observed

- **Verbal or finger counting:** Noticed more in government school students compared to private school students.
  - In lower grade: finger counting and in the later grades, seen counting in their head verbally.
  - More students used finger counting for subtraction than addition
  - Some students were using a minimum number counting strategy. For example, if they have to count 3 + 8, they are counting on 9, 10 and 11 from 8 instead of counting on from 3, in their head to answer as 11.
- **Count all strategy:** More among government school students and a few students in bottom 10 percentile in the private schools. (To answer for say 5 + 3, they would count 5 fingers first, 3 more and then count all the fingers to answer as 8.)
- **Counting on strategy**: Most of the private school students who used finger or verbal counting seem to be applying count on strategy (e.g. 7 + 3 as 7 in mind and counting on 8, 9 and 10 instead of counting 7 fingers and then 3 fingers to count all fingers finally).
- **Doubling:** Some students in private schools and govt. schools seem to be comfortable with multiplication facts and they converted doubling fact to a multiplication fact (e.g. 8 + 8 as 2 times 8). This strategy is noticed in some students in 2nd and 3rd quartile private school students as well.
- **Decomposition:** Mostly top 10 percentile private school students (students with a good score of number corrects per minute in the two fact fluency timed tests) seem to be applying this strategy of decomposing the given fact to an easier fact for which they have developed automaticity. This doesn't seem to be a traditionally taught method or logic. The students seem to be developing such strategies on their own. Some examples are given below.
  - Making it to pairs of 10: 3 + 8 as 1 + 10 (as 2 + 8 = 10); 9 + 3 as 10 + 2;
  - converting into easier facts: 5 + 7 as 10 + 2 (5+5+2); 7 + 9 as 6 + 10; 8 + 9 as 2×8 + 1 (doubling)
- **Count on for subtraction:** Students are using finger/verbal counting more for subtraction both in govt. and private schools. But some students are using count on strategy. E.g. to answer 12 - 8, they may count 9, 10, 11 and 12 and then answer it as 4.
- In both private and government schools, very rarely the relevant addition fact is used to answer the subtraction fact.
  E.g. 12 - 4 can be answered using 8 + 4 = 12 which can be seen in adults.

DISCUSSION

- The strategies used by students to answer the addition and subtraction facts also indicate their levels of fluency and help validate the levels indicated in ncpm.
- A student typically used any of the mental strategies, counting on figures or using marks on paper to count to answer for different facts.
- Students tend to count on fingers more for subtraction facts than addition facts
- Found **mix of strategies used by a single student** in general varying as per the addition/subtraction fact rather than the same strategy used consistently for all the facts.

Recommendations:

- Based on the findings we propose educators should focus on developing fact fluency especially among government school students and include measuring fluency in formative assessments. Fact fluency does not get build on its own.
- Research on instructional practices to develop fact fluency effectively and earlier in grades especially among government schools is proposed (students are not developing fluency on their own).
- In future, measures which focus on evaluating method (strategy) used to answer a fact can be developed for use in formative assessments in classrooms